vessels must be in line and the rate of approach the maximum. Hence if A does m knots between the signals and B does n knots the rule holds that $d_0 = d_1 + (m+n) = d_2 + 2(m+n)$, etc. Again, if one vessel is overtaking the other, both being on the same course, the rate of approach is a maximum if collision is threatened and $d_0 = d_1 + (m-n) = d_2 + 2(m-n)$, etc., when A is overtaking B. If this arithmetical relation is not found to hold for the successive determinations of distance, and if the amount of departure from equality increases with each observation, there is safety. The vessels are not proceeding in the same line.

On the Oxy-hydrogen Flame Spectrum of Iron.

By Sir Norman Lockyer, K.C.B., Hon. D.Sc., LL.D., F.R.S., and H. E. Goodson, A.R.C.Sc.

(Received December 6, 1915.)

A few lines in the visual region of the oxy-hydrogen flame spectrum of iron were recorded in 1887 by Sir Norman Lockyer.* This list was supplemented in 1893† by a map showing all the then known flame lines of iron in the photographic region. In the following year Hartley‡ published his researches on "Flame Spectra at High Temperature," including a list of lines in the spectrum obtained by heating compounds of iron in the oxy-hydrogen flame. This list extends from λ 5927·7 to λ 3021·1.

Flame spectra of iron have been studied in great detail by de Watteville, working alone or in collaboration with Hemsalech. In particular they have published§ an extensive list of wave-lengths and intensities of lines observed in the oxy-hydrogen flame fed by a current of oxygen previously passed through a globe in which an electric spark was being maintained between iron poles.

It may also be mentioned that reproductions of photographs of the iron flame spectra have been published in the atlases of Hagenbach and Konen (1905) and Eder and Valenta (1911).

Some preliminary results obtained from a spectrum of iron burning in the

* 'Roy. Soc. Proc.,' vol. 43, p. 120 (1887).

† 'Phil. Trans.,' A, vol. 184, pp. 675–726, Plate 28 (1893).

‡ 'Phil. Trans.,' A, vol. 185, pp. 199–202 (1894).

§ 'Comptes Rendus,' vol. 146, p. 964 (1908).

oxy-hydrogen flame were communicated to the Royal Society by Sir Norman Lockyer* in 1911. The present paper contains an account of further work on the same spectrum carried out on glass negatives taken from a copy, also on glass, of a spectrogram secured with the 3-inch Cooke prism spectrograph of the Solar Physics Observatory, then at South Kensington. In this spectrum 64 lines of iron have been identified between $\lambda\lambda$ 3856·52 and 5615·88; 15 of these lines have not hitherto been recorded in the iron flame spectrum.

Identification of the lines was effected by measuring approximately portions of the spectrum in turn under the microscope of a star plate measuring machine. The measures were reduced by the Cornu-Hartmann interpolation formula and the means compared with the list of arc lines of iron given in the first part of Eder and Valenta's† atlas and also with the lines in the list given by King.‡ Eder and Valenta's wave-lengths have been adopted where corresponding lines appeared in their list. In those cases where they give no line Rowland's solar wave-lengths have been taken. In a few cases it was impossible to discriminate between the components of pairs of lines in Eder and Valenta's list, and the measured wave-length was consequently retained.

The flame spectrum was next compared with similar copies of an iron arc spectrum. Two well-marked groups of lines were at once distinguished, the one consisting of lines stronger, *i.e.* relatively more conspicuous, in the flame than in the arc, including a number of lines which did not appear to be represented in the arc spectrum employed, and the other containing lines weaker, *i.e.* relatively less conspicuous, in the flame than in the arc. The remaining lines, nearly equal in both spectra, but which were all described as doubtfully weakened in the flame, formed an intermediate group. All the lines have been placed in one or other of these categories, designated respectively:—

Group A. Lines stronger in flame than arc.
„ B. „ weaker „ „
„ C. „ nearly equal in both sources.

The results of this comparison are given in Table I, which follows.

A small number of lines in this list are recorded absent from the comparison spectrum (arc) and a still smaller number—two only—do not appear in the list of lines given by Eder and Valenta for the arc spectrum of iron. All, however, are recorded by Kayser§ among the arc lines of iron,

* 'Roy. Soc. Proc.,' A, vol. 86, pp. 78–80 (1911).
† 'Atlas Typischer Spektren,' Vienna, 1911.
‡ 'Astrophys. Journ.,' vol. 37, p. 252 *et seq.* (1913).
§ 'Handbuch der Spectroscopie,' vol. 6, p. 901 (1912).

Table I.—Behaviour of Iron Lines in Flame and Arc.

Flame lines of iron. Hill Observatory. λλ.	Int.	Behaviour in flame spectrum compared with that in arc.	Group.	Flame lines of iron. Hill Observatory. λλ.	Int.	Behaviour in flame spectrum compared with that in arc.	Group.
3856·52	3–4	Much strengthened	A	4325·94	5–6	? Weakened	C
3860·05	4–5	,, ,,	A	4376·10	4–5	Strengthened	A
3865·67	0–1	Much weakened	B	4383·72	5–6	? Weakened	C
3872·70	0–1	,, ,,	B	4404·93	2–3	Weakened	B
3878·17	0–1	Very much weakened*	B	4415·30	2	Much weakened	B
3878·72	4–5	Strengthened	A	4427·49	3–4	Much strengthened	A
3886·43	5–6	,,	A	4435·321	0–1	No line in this position	A
3887·19	0–1	Weakened	B			in S.K. arc	
3895·80	4	Strengthened	A	4461·84	2–3	Much strengthened	A
3899·85	5	,,	A	4482·39	1–2	Strengthened	A
3903·10	0–1	Much weakened	B	4489·93	0–1	Much strengthened	A
3906·62	2	Strengthened	A	4957·79	0–1	Very much weakened	B
3920·40	4	,,	A	4994·32	0–1	Not visible in S.K. arc	A
3923·06	5	,,	A	5012·25	1	,, ,,	A
3928·07	5	,,	A	5041·26	0–1	,, ,,	A
3930·49	5	,,	A	5051·83	0–1	,, ,,	A
3969·41	1–2	Much weakened	B	5110·57	2	,, ,,	A
4005·42	1–2	,, ,,	B	5166·45	1–2	Strengthened	A
4045·98	7	? Weakened	C	5167·68	0–1	Very much weakened	B
4063·76	5–6	,,	C	5169·07	1–2	Strengthened	A
4071·90	4–5	Slightly weakened	B	5171·78	0–1	Not visible in S.K. arc	A
4132·25	1	Much weakened	B	5204·77	0–1	,, ,,	A
4143·8	2	Weakened	B	5227·2	1	? Weakened	C
4202·20	1–2	,,	B	5269·72	4–5	Strengthened	A
4206·862	0–1	Much strengthened	A	5328·24	3	,,	A
4216·33	2–3	Greatly strengthened	A	5371·73	2	,,	A
4250·95	2	Weakened	B	5397·34	1–2	,,	A
4260·66	0–1	Very much weakened	B	5405·99	1–2	Slightly strengthened	A
4271·93	4–5	? Weakened	C	5429·91	1–2	Strengthened	A
4291·63	0–1	Doubtfully present in	A	5434·74	0–1	,,	A
		S.K. arc.		5447·13	1–2	,,	A
4294·29	0–1	Weakened	B	5455·83	1	,,	A
4308·09	5–6	? Weakened	C	5615·88	1	Very much weakened	B

* In the arc λ 3878·17 is slightly stronger than λ 3878·72, but in the flame λ 3878·72 is much the stronger, λ 3878·17 having very nearly vanished. Thus this pair affords a very good example of intensity inversion.

and all appear also as solar lines in Rowland's preliminary Tables of solar wave-lengths.

A detailed comparison has been made with the results recently published by King* in an important memoir on the variation with temperature of the electric furnace spectrum of iron. It has been found that the lines recorded in the flame include all the stronger lines observed by King in the low-temperature furnace. All lines of iron of an intensity in that source superior to 6 (maximum 25) have been recorded in the flame spectrum, and

* 'Astrophys. Journ.,' vol. 37, p. 239 (1913).

whilst the great majority of the lines of less strength have not been included, yet a few weaker lines have been recorded as very faintly represented. Twelve lines, including seven of King's Class IA and four of Class IB, stronger in the low-temperature furnace than the above have not been recorded in the flame; seven of these, however, fall between λλ 4994 and 5507, where the iron flame spectrum is much obscured by continuous spectrum and bands.

These differences indicate that the oxy-hydrogen flame spectrum of iron obtained by burning the metal represents a temperature level somewhat above the low-temperature furnace conditions employed by King.

The lines of Group A form part, with only one exception, of King's Classes IA and IB—the typical low-temperature classes. The lines of Groups B and C, on the other hand, are principally included in King's Class II, a few being placed in Classes III and IV. The flame lines having the greatest strength are nearly all of King's Class II, and have been placed in Group C. The lines of King's Class IA, seven of which are herein recorded in the flame, are all very weak arc lines, their arc intensities ranging from 2 to 4 (maximum 60, King). They are all weak in the flame.

The flame spectrum has also been compared with the list of solar iron lines and their behaviour in sunspot spectra given by Adams,* in his valuable paper on the core and flame spectra of the iron arc. The lines herein described as Group A flame lines of iron, which appear in the list given by Adams, are nearly all about twice as strong in the flame of the electric arc as in its core, and none are less than one and a half times as strong. Thus, of course, the lines of Group A are strengthened in sunspot spectra. The lines of Groups B and C, on the other hand, are either unchanged or reduced in intensity.

The stellar behaviour of the flame lines has been investigated by comparison with the observations of Miss Maury† on the intensities of the so-called "solar lines" in a number of typical stellar spectra.

Although it is recognised that the level of initial appearance in stellar spectra cannot be precisely stated, yet the occurrence of lines probably involving the iron flame lines of Group A seems limited by the Sirian stars, and they are quite definitely established at the level of Procyon. Lines of Groups B and C, however, seem to range as high as β Persei (Alg.), and are well established in α Canis Majoris.

Homogeneous data are available for nine lines of Group A between

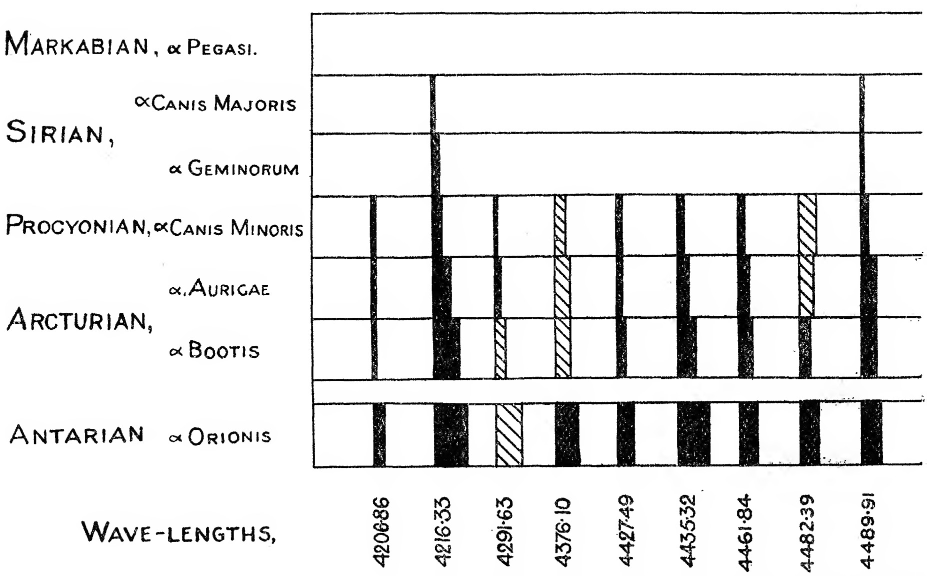
* 'Astrophys. Journ.,' vol. 30, p. 98 (1909).

† 'Harv. Coll. Obs. Ann.,' vol. 28, Part I (1897).

λλ 4206·9 and 4489·4 in regard to six stellar spectra representing five successive levels of the Kensington classification and six groups in the classification of Miss Maury. The behaviour shown by these nine lines is uniform. Although only two of them have been recorded in the Sirian stage, all are possibly represented in α Canis Minoris (Proc.) and the succeeding lower stages, and in all cases except one where the quoted intensities refer to a pair of lines in α Canis Majoris which are separated at lower levels, an increase of intensity is shown in passing from the higher level to that of α Orionis. In nearly every case the increase is uniformly progressive.

The stellar behaviour of the Group A lines referred to above is shown in the accompanying diagram (fig. 1). The stellar intensities are represented proportionately by the width of the lines on the several horizons.

BEHAVIOUR OF GROUP A FLAME LINES OF IRON IN STARS.



THE SHADED PORTIONS INDICATE THAT THE STELLAR LINE INCLUDES AN ADJACENT LINE.

FIG. 1.

In the region on the less refrangible side of λ 4489·9 data are only available for three stellar stages, and, except in two cases, the intensities of the lines show an increase in passing from the hotter to the cooler stellar level

With regard to the two outstanding lines Keeler* has recorded that they are both stronger in α Orionis than in the solar spectrum.

The stellar behaviour of the iron flame lines of Group A is thus exactly in accord with their behaviour in the sunspot spectra as compared with the Fraunhöferic, and also just what would be expected from their laboratory behaviour.

The Theory of the Helmholtz Resonator.

By Lord Rayleigh, O.M., F.R.S.

(Received December 15, 1915.)

The ideal form of Helmholtz resonator is a cavernous space, almost enclosed by a thin, immovable wall, in which there is a small perforation establishing a communication between the interior and exterior gas. An approximate theory, based upon the supposition that the perforation is small, and consequently that the wave-length of the aërial vibration is great, is due to Helmholtz,† who arrived at definite results for perforations whose outline is circular or elliptic. A simplified, and in some respects generalised, treatment was given in my paper on "Resonance."‡ In the extreme case of a wave-length sufficiently great, the kinetic energy of the vibration is that of the gas near the mouth as it moves in and out, much as an incompressible fluid might do, and the potential energy is that of the almost uniform compressions and rarefactions of the gas in the interior. The latter is a question merely of the volume S of the cavity and of the quantity of gas which has passed, but the calculation of the kinetic energy presents difficulties which have been only partially overcome. In the case of simple apertures in the thin wall (regarded as plane), only circular and elliptic forms admit of complete treatment. The mathematical problem is the same as that of finding the electrostatic *capacity* of a thin conducting plate having the form of the aperture, and supposed to be situated in the open.

The project of a stricter treatment of the problem, in the case of a

* Quoted "Spectra of Stars of Secchi's Fourth Type," 'Pub. Yerkes Obs.,' vol. 2, p. 371 (1903).

† 'Crelle Jl. Math.,' vol. 57 (1860).

‡ 'Phil. Trans.,' vol. 161, p. 77 (1870); 'Scientific Papers,' vol. 1, p. 33. Also 'Theory of Sound,' ch. xvi.